Yellowknife

1566 km

67.3°

Vancouver

1870 km

Acute Triangle Trigonometry

▶ LEARNING GOALS

You will be able to develop your spatial sense by

- Using the sine law to determine side lengths and angle measures in acute triangles
- Using the cosine law to determine side lengths and angle measures in acute triangles
- Solving problems that can be modelled using acute triangles

?

Winnipeg

? Who would need to know an accurate distance from Yellowknife to Winnipeg?

Why can you not use sine, cosine, or tangent ratios directly to determine the distance?

Getting Started

Lacrosse Trigonometry

Field lacrosse, Canada's national sport, originated with First Nations peoples, probably in central North America. Hundreds of years ago, both the number of players and the size of the field were much greater than in the modern game. The Iroquois peoples of what is now southern Ontario and western New York may have been the first to limit the number of players to 12 or 15.

Daniel is about to take a shot at a field lacrosse net. He estimates his current position, as shown below.

? **How can you use these estimates to determine the width of the net?**

A. Does Daniel's position form a right triangle with the goalposts?

B. A primary trigonometric ratio cannot be used to determine the width of the net directly. Explain why.

C. Copy the triangle that includes Daniel's position in the diagram above. Add a line segment so that you can determine a height of the triangle using trigonometry.

D. Determine the height of the triangle using a primary trigonometric ratio.

E. Create a plan that will allow you to determine the width of the lacrosse net using the two right triangles you created.

F. Carry out your plan to determine the width of the net.

This team from Caughnawaga toured England in 1867, demonstrating the game. As a result of the tour, lacrosse clubs were established in England. The game then spread around the world.

The Canadian team won the bronze medal at the 2009 Women's Lacrosse World Cup in Prague, Czech Republic.

WHAT DO You Think?

Decide whether you agree or disagree with each statement. Explain your decision.

1. When using proportional reasoning, if you apply the same operation, using the same number, to all the terms, the ratios remain equivalent.

2. If you know the measures of two angles and the length of any side in an acute triangle, you can determine all the other measurements.

3.1

Exploring Side–Angle Relationships in Acute Triangles

Inukshuks can have many meanings. Some inukshuks direct travel, some indicate rich fishing or hunting areas, and some warn of danger.

GOAL
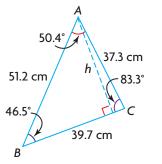
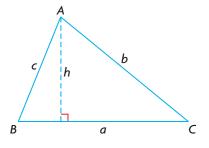Explore the relationship between each side in an acute triangle and the sine of its opposite angle.

EXPLORE the Math

As they explore the North, the Inuit leave stone cairns, called inukshuks, as markers for those who follow in their path.

You have used the primary trigonometric ratios to determine side lengths and angle measures in right triangles. Can you use primary trigonometric ratios to determine unknown sides and angles in all acute triangles?

Choose one of the triangles below. The first triangle is a scale diagram of the side of the inukshuk shown. The second triangle represents a general acute triangle.
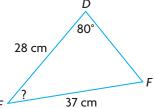
? What are two equivalent expressions that represent the height of △ABC?

Reflecting

A. Find a classmate who chose a different triangle than you did. Compare each set of expressions. How are they the same and how are they different?

B. If you drew the height of △ABC from a different vertex, how would the expressions for that height be different? Explain.

C. Create an equation using the expressions you created in part A. Show how your equation can be written so that each ratio in the equation involves a side and an angle. Repeat for the expressions you described in part B.

D. Explain how you could determine the measure of ∠E in this acute triangle.

7152_FM11SB_C3_L3_ALB_pp130-131.indd   130   9/23/10   4:31:18 PM
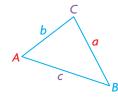
In Summary

Key Idea

- The ratios of $\dfrac{\text{length of opposite side}}{\sin(\text{angle})}$ are equivalent for all three side–angle pairs in an acute triangle.
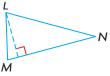
Need to Know

- In an acute triangle, $\triangle ABC$,

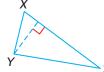$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

FURTHER *Your Understanding*

1. For each acute triangle,
   i) copy the triangle and label the sides.
   ii) write two expressions for the height of each triangle, and use your expressions to create equivalent ratios.

   a)

   b)

2. i) Sketch a triangle that corresponds to each equation below.
   ii) Solve for the unknown side length or angle measure. Round your answer to one decimal place.

   a) $\dfrac{w}{\sin 50°} = \dfrac{8.0}{\sin 60°}$

   c) $\dfrac{6.0}{\sin M} = \dfrac{10.0}{\sin 72°}$

   b) $\dfrac{k}{\sin 43°} = \dfrac{9.5}{\sin 85°}$

   d) $\dfrac{12.5}{\sin Y} = \dfrac{14.0}{\sin 88°}$

3. Michel claims that if x and y are sides in an acute triangle, then:

   $$x \sin Y = y \sin X$$

   Do you agree or disagree? Justify your decision.

4. If you want to determine an unknown side length or angle measure in an acute triangle, what is the minimum information that you must have?

5. Do you think the ratios of $\dfrac{\text{opposite side}}{\sin(\text{angle})}$ are equivalent for all three side–angle pairs in a right triangle? Construct two right triangles, and measure their sides and angles. Use your measurements to test your conjecture.

> **Communication | Tip**
>
> The expression $x \sin Y$ is a product. It is equivalent to $x(\sin Y)$.
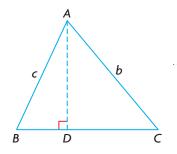
Proving and Applying the Sine Law

EXPLORE...

- The angles in an acute triangle measure 40°, 55°, and 85°. Could two of the side lengths be 5 cm and 4 cm? Explain.

GOAL

Explain the steps used to prove the sine law. Use the law to solve triangles.

INVESTIGATE the Math

In Lesson 3.1, you discovered a side–angle relationship in acute triangles. Before this relationship can be used to solve problems, it must be proven to work in all acute triangles. Consider Ben's **proof**:

Step 1
I drew an acute triangle with height AD.

In $\triangle ABD$,

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AD}{c}$$

$$c \sin B = AD$$

In $\triangle ACD$,

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{AD}{b}$$

$$b \sin C = AD$$

Step 2
I wrote equations for the sine of $\angle B$ and the sine of $\angle C$ in the two right triangles.
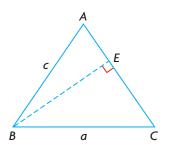
$$c \sin B = b \sin C$$

$$\frac{c \sin B}{\sin C} = b$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

Step 3
I set the expressions for AD equal to each other.

In $\triangle ABE$,

$\sin A = \dfrac{BE}{c}$

$c \sin A = BE$

In $\triangle CBE$,

$\sin C = \dfrac{BE}{a}$

$a \sin C = BE$

$c \sin A = a \sin C$

$c = \dfrac{a \sin C}{\sin A}$

$\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

? How can you improve Ben's explanation of his proof?

A. Work with a partner to explain why Ben drew height AD in step 1.

B. In step 2, he created two different expressions that involved AD. Explain why.

C. Explain why he was able to set the expressions for AD equal in step 3.

D. Explain what Ben did to rewrite the equation in step 3.

E. In steps 4 and 5, Ben drew a different height BE and repeated steps 2 and 3 for the right triangles this created. Explain why.

F. Explain why he was able to equate all three ratios in step 6 to create the **sine law**.
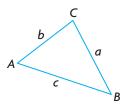
Reflecting

G. Why did Ben not use the cosine ratio or tangent ratio to describe the heights of his acute triangle?

H. If Ben drew a perpendicular line segment from vertex C to side AB, which pair of ratios in the sine law do you think he could show to be equal?

I. Why does it make sense that the sine law can also be written in the form $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$?

sine law

In any acute triangle,

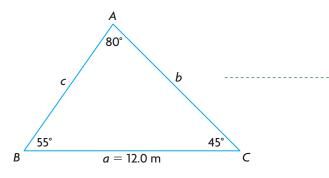$$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$$

APPLY the Math

EXAMPLE 1 | Using reasoning to determine the length of a side

A triangle has angles measuring 80° and 55°. The side opposite the 80° angle is 12.0 m in length. Determine the length of the side opposite the 55° angle to the nearest tenth of a metre.
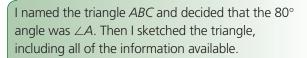
Elizabeth's Solution

I named the triangle *ABC* and decided that the 80° angle was $\angle A$. Then I sketched the triangle, including all of the information available.

I knew that the third angle, $\angle C$, had to measure 45°, because the angles of a triangle add to 180°. I needed to determine *b*.

Since the triangle does not contain a right angle, I couldn't use the primary trigonometric ratios.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{12.0}{\sin 80°} = \frac{b}{\sin 55°}$$

$$\sin 55°\left(\frac{12.0}{\sin 80°}\right) = \sin 55°\left(\frac{b}{\sin 55°}\right)$$

$$\sin 55°\left(\frac{12.0}{\sin 80°}\right) = b$$

$$9.981... = b$$

I could use the sine law if I knew an opposite side–angle pair, plus one more side or angle in the triangle. I knew *a* and $\angle A$ and I wanted to know *b*, so I related *a*, *b*, sin *A*, and sin *B* using $\frac{a}{\sin A} = \frac{b}{\sin B}$. Since *b* was in the numerator, I could multiply both sides by sin 55° to solve for *b*.

The length of *AC* is 10.0 m.

I rounded to the nearest tenth. It made sense that the length of *AC* is shorter than the length of *BC*, since the measure of $\angle B$ is less than the measure of $\angle A$.
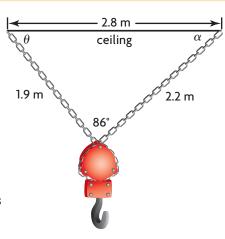
Your Turn

Using $\triangle ABC$ above, determine the length of *AB* to the nearest tenth of a metre.

EXAMPLE **2** | Solving a problem using the sine law

Toby uses chains attached to hooks on the ceiling and a winch to lift engines at his father's garage. The chains, the winch, and the ceiling are arranged as shown. Toby solved the triangle using the sine law to determine the angle that each chain makes with the ceiling to the nearest degree. He claims that $\theta = 40°$ and $\alpha = 54°$. Is he correct? Explain, and make any necessary corrections.

Sanjay's Solution

I know Toby's calculations are incorrect, since α must be the smallest angle in the triangle.

> In any triangle, the shortest side is across from the smallest angle. Since 1.9 m is the shortest side, $\alpha < \theta$. Toby's values do not meet this condition.

$$\frac{\sin \alpha}{1.9} = \frac{\sin 86°}{2.8}$$

> To correct the error, I used the sine law to determine α.

$$1.9\left(\frac{\sin \alpha}{1.9}\right) = 1.9\left(\frac{\sin 86°}{2.8}\right)$$

$$\sin \alpha = 1.9\left(\frac{\sin 86°}{2.8}\right)$$

$$\sin \alpha = 0.6769...$$
$$\alpha = \sin^{-1}(0.6769...)$$
$$\alpha = 42.603...°$$

> I multiplied both sides by 1.9 to solve for sin α. Then I evaluated the right side of the equation.

$$\theta = 180° - 86° - 42.603...°$$
$$\theta = 51.396...°$$

> I used the fact that angles in a triangle add to 180° to determine θ.

Toby was incorrect. The correct measures of the angles are:

$$\alpha \doteq 43°$$
and
$$\theta \doteq 51°$$

> My determinations are reasonable, because the shortest side is opposite the smallest angle.
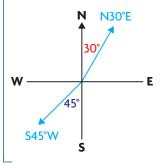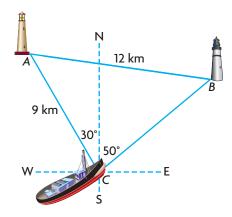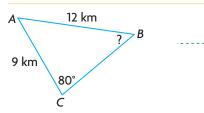
EXAMPLE 3 | Using reasoning to determine the measure of an angle

The captain of a small boat is delivering supplies to two lighthouses, as shown. His compass indicates that the lighthouse to his left is located at N30°W and the lighthouse to his right is located at N50°E. Determine the compass direction he must follow when he leaves lighthouse *B* for lighthouse *A*.

Anthony's Solution

I drew a diagram. I labelled the sides of the triangle I knew and the angle I wanted to determine.

$$\frac{\sin B}{AC} = \frac{\sin C}{AB}$$

I knew *AC*, *AB*, and ∠*C*, and I wanted to determine ∠*B*. So I used the sine law that includes these four quantities.

I used the proportion with sin *B* and sin *C* in the numerators so the unknown would be in the numerator.

$$\frac{\sin B}{9} = \frac{\sin 80°}{12}$$

$$9\left(\frac{\sin B}{9}\right) = 9\left(\frac{\sin 80°}{12}\right)$$

$$\sin B = 9\left(\frac{\sin 80°}{12}\right)$$

$$\sin B = 0.7386...$$

$$\angle B = \sin^{-1}(0.7386...)$$
$$\angle B = 47.612...°$$

> I substituted the given information and then solved for sin B.
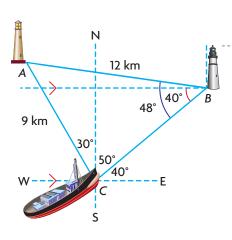
The Fisgard Lighthouse in Victoria, British Columbia, was the first lighthouse built on Canada's west coast and is still in operation today.

The measure of $\angle B$ is 48°.

> The answer seems reasonable. $\angle B$ must be less than 80°, because 9 km is less than 12 km.

> I drew a diagram and marked the angles I knew. I knew east-west lines are all parallel, so the alternate interior angle at B must be 40°.

The captain must head N82°W from lighthouse B.

> The line segment from lighthouse B to lighthouse A makes an 8° angle with west-east. I subtracted this from 90° to determine the direction west of north.

Your Turn

In $\triangle ABC$ above, CB is about 9.6 km. Use the sine law to determine $\angle A$. Verify your answer by determining the sum of the angles.
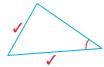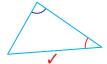
In Summary

Key Idea
- The sine law can be used to determine unknown side lengths or angle measures in acute triangles.

Need to Know
- You can use the sine law to solve a problem modelled by an acute triangle when you know:
  - two sides and the angle opposite a known side.

  - two angles and any side.

- If you know the measures of two angles in a triangle, you can determine the third angle because the angles must add to 180°.

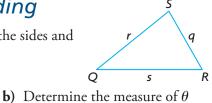- When determining side lengths, it is more convenient to use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

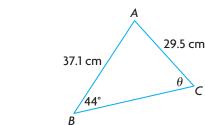- When determining angles, it is more convenient to use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

CHECK *Your Understanding*
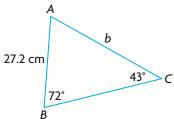
1. Write three equivalent ratios using the sides and angles in the triangle at the right.

2. **a)** Determine length b to the nearest tenth of a centimetre.

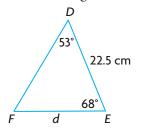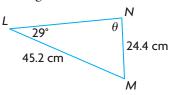   **b)** Determine the measure of θ to the nearest degree.
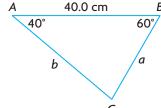
PRACTISING

3. Determine the indicated side lengths to the nearest tenth of a unit and the indicated angle measures to the nearest degree.
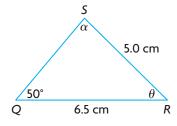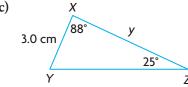
a)

b)

c)

d)

e)
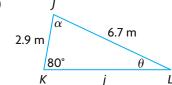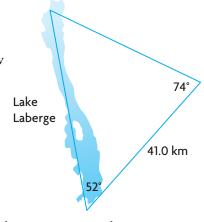
f)

4. Scott is studying the effects of environmental changes on fish populations in his summer job. As part of his research, he needs to know the distance between two points on Lake Laberge, Yukon. Scott makes the measurements shown and uses the sine law to determine the lake's length as 36.0 km.
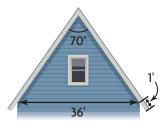
a) Agathe, Scott's research partner, says that his answer is incorrect. Explain how she knows.

b) Determine the distance between the two points to the nearest tenth of a kilometre.

5. An architect designed a house and must give more instructions to the builders. The rafters that hold up the roof are equal in length. The rafters extend beyond the supporting wall as shown. How long are the rafters? Express your answer to the nearest inch.

The acidity of northern lakes may be affected by acid rain and snow caused by development.
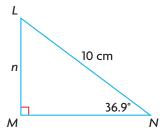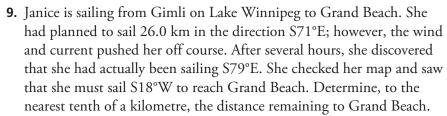
6. Draw a labelled diagram for each triangle. Then determine the required side length or angle measure.

    a) In $\triangle SUN$, $n = 58$ cm, $\angle N = 38°$, and $\angle U = 72°$. Determine the length of side u to the nearest centimetre.

    b) In $\triangle PQR$, $\angle R = 73°$, $\angle Q = 32°$, and $r = 23$ cm. Determine the length of side q to the nearest centimetre.

    c) In $\triangle TAM$, $t = 8$ cm, $m = 6$ cm, and $\angle T = 65°$. Determine the measure of $\angle M$ to the nearest degree.

    d) In $\triangle WXY$, $w = 12.0$ cm, $y = 10.5$ cm, and $\angle W = 60°$. Determine the measure of $\angle Y$ to the nearest degree.

7. In $\triangle CAT$, $\angle C = 32°$, $\angle T = 81°$, and $c = 24.1$ m. Solve the triangle. Round angles to the nearest degree and sides to the nearest tenth of a metre.

8. a) Determine the value of n to the nearest tenth using
      i) a primary trigonometric ratio.
      ii) the sine law.

    b) Explain why your answers for part a) are the same. Do others in your class agree with your explanation?

9. Janice is sailing from Gimli on Lake Winnipeg to Grand Beach. She had planned to sail 26.0 km in the direction S71°E; however, the wind and current pushed her off course. After several hours, she discovered that she had actually been sailing S79°E. She checked her map and saw that she must sail S18°W to reach Grand Beach. Determine, to the nearest tenth of a kilometre, the distance remaining to Grand Beach.

    a) Draw a diagram of this situation, then compare it with a classmate's. Make any adjustments needed in your diagrams.

    b) Solve the problem.

Gimli, Manitoba, is a fishing village with a rich Icelandic heritage.

10. A telephone pole is supported by two wires on opposite sides. At the top of the pole, the wires form an angle of 60°. On the ground, the ends of the wires are 15.0 m apart. One wire makes a 45° angle with the ground.

    a) Draw a diagram of this situation, then compare it with a classmate's.

    b) How long are the wires, and how tall is the pole? Express your answers to the nearest tenth of a metre.

11. In $\triangle PQR$, $\angle Q = 90°$, $r = 6$, and $p = 8$. Explain two different ways to determine the measure of $\angle P$. Share your explanation with a partner. How might you improve your explanation?

12. Stella decided to ski to a friend's cabin. She skied 10.0 km in the direction N40°E. She rested, then skied S45°E and arrived at the cabin. The cabin is 14.5 km from her home, as the crow flies. Determine, to the nearest tenth of a kilometre, the distance she travelled on the second leg of her trip.
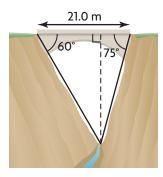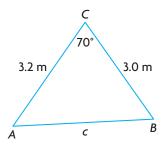
13. A bridge has been built across a gorge. Jordan wants to bungee jump from the bridge. One of the things she must know, to make the jump safely, is the depth of the gorge. She measured the gorge as shown. Determine the depth of the gorge to the nearest tenth of a metre.

14. Sketch an acute triangle.
- **a)** List three pieces of information about the triangle's sides and angles that would allow you to solve for all the other side lengths and angle measures of the triangle.
- **b)** List three pieces of information about the triangle's sides and angles that would not allow you to solve the triangle.

15. Jim says that the sine law cannot be used to determine the length of side c in $\triangle ABC$. Do you agree or disagree? Explain.

Closing

16. Suppose that you know the length of side p in $\triangle PQR$, as well as the measures of $\angle P$ and $\angle Q$. What other sides and angles could you determine? Explain to a classmate how you would determine these measurements.

Extending

17. In $\triangle ABC$, $\angle A = 58°$, $\angle C = 74°$, and $b = 6$. Determine the area of $\triangle ABC$ to one decimal place.

18. An isosceles triangle has two sides that are 10.0 cm long and two angles that measure 50°. A line segment bisects one of the 50° angles and ends at the opposite side. Determine the length of the line segment to the nearest tenth of a centimetre.

19. Use the sine law to write a ratio that is equivalent to each expression for $\triangle ABC$.
- **a)** $\dfrac{\sin A}{\sin B}$
- **b)** $\dfrac{a}{c}$
- **c)** $\dfrac{a \sin C}{c \sin A}$
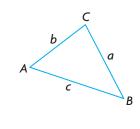
FREQUENTLY ASKED *Questions*

Q: **What is the sine law, and what is it used for?**

A: The sine law describes the relationship between the sides and their opposite angles in a triangle.

In $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The sine law can be used to determine unknown side lengths and angle measures in acute and right triangles.

Q: **When can you use the sine law?**

A: Use the sine law if you know any three of these four measurements: two sides and their opposite angles. The sine law allows you to determine the unknown length or angle measure. If you know any two angles in a triangle, you can determine the third angle.

For example, the sine law can be used to determine the length of AB in the triangle at the right.
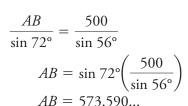
Side AB is opposite $\angle C$, which measures 72°.

Side AC is of length 500 m and is opposite $\angle B$.

$\angle B = 180° - 52° - 72°$

$\angle B = 56°$

Use the sine law to write an equation and solve for AB.

$$\frac{AB}{\sin 72°} = \frac{500}{\sin 56°}$$

$$AB = \sin 72°\left(\frac{500}{\sin 56°}\right)$$

$$AB = 573.590...$$

The length of AB is about 574 m.

PRACTISING

Lesson 3.1

1. Write the equivalent sine law ratios for acute triangle *XYZ*.

2. **a)** Sketch an acute triangle that illustrates the relationship described in the equation below.
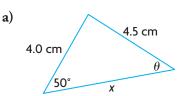$$\frac{x}{\sin 60°} = \frac{10}{\sin 80°}$$
   b) Determine the value of *x* to the nearest tenth.

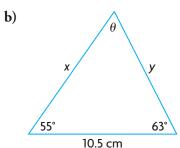3. $\triangle DEF$ is an acute triangle. Nazir claims that
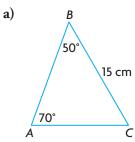$$\frac{d}{\sin F} = \frac{f}{\sin D}$$
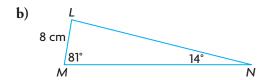   Do you agree or disagree? Explain.

Lesson 3.2

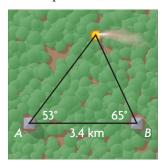4. Determine the values indicated with variables to the nearest tenth of a unit.

   a)

   4.0 cm 4.5 cm
   50° *x* θ

   b)

   θ
   x *y*
   55° 63°
   10.5 cm

5. Solve each triangle. Round all answers to the nearest tenth of a unit.

   a)

   B
   50°
   15 cm
   70°
   A *C*

---

b)

L
8 cm
81° 14°
M *N*
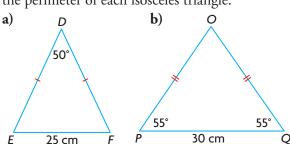
6. In $\triangle XYZ$, the values of *x* and *z* are known. What additional information do you need to know if you want to use the sine law to solve the triangle?

7. Two Jasper National Park rangers in their fire towers spot a fire.

   53° 65°
   A 3.4 km *B*

   a) Which tower is closer to the fire? Explain.
   b) Determine the distance, to the nearest tenth of a kilometre, from this tower to the fire.

8. As Chloe and Ivan are paddling north on Lac La Ronge in Saskatchewan, they notice a campsite ahead, at N52°W. They continue paddling north for 800 m, which takes them past the campsite. The campsite is then at S40°W. How far away, to the nearest metre, is the campsite from their position at the second sighting?

9. Determine, to the nearest tenth of a centimetre, the perimeter of each isosceles triangle.

   a)

   D
   50°
   E 25 cm *F*

   b)

   O
   55° 55°
   P 30 cm *Q*

Mid-Chapter Review **143**

3.3 Proving and Applying the Cosine Law

YOU WILL NEED
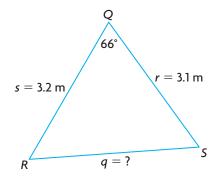
- ruler
- protractor
- calculator

EXPLORE...

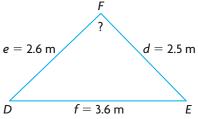- One side of a right triangle is 8 cm. One angle is 50°. What could the other side lengths be?

GOAL

Explain the steps used to prove the cosine law. Use the cosine law to solve triangles.

INVESTIGATE the Math

The sine law cannot always help you determine unknown angle measures or side lengths. Consider these triangles:
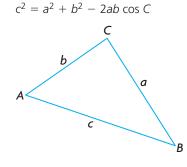
where $\dfrac{3.1}{\sin R} = \dfrac{3.2}{\sin S} = \dfrac{q}{\sin 66°}$

where $\dfrac{\sin E}{2.6} = \dfrac{\sin D}{2.5} = \dfrac{\sin F}{3.6}$
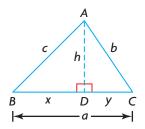
There are two unknowns in each pair of equivalent ratios, so the pairs cannot be used to solve for the unknowns. Another relationship is needed. This relationship is called the **cosine law**, and it is derived from the Pythagorean theorem.

Before this relationship can be used to solve problems, it must be proven to work in all acute triangles. Consider Heather's proof of the cosine law:

cosine law

In any acute triangle,

$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

Step 1
I drew an acute triangle *ABC*. Then I drew the height from *A* to *BC* and labelled the intersection point as point *D*. I labelled this line segment *h*. I labelled *BD* as *x* and *DC* as *y*.

$$h^2 = c^2 - x^2$$
$$h^2 = b^2 - y^2$$

Step 2
I wrote two different expressions for h^2.

$$c^2 - x^2 = b^2 - y^2$$
$$c^2 = x^2 + b^2 - y^2$$

Step 3
I set the two expressions equal to each other and solved for c^2.

$$x = a - y, \text{ so}$$
$$c^2 = (a - y)^2 + b^2 - y^2$$
$$c^2 = a^2 - 2ay + y^2 + b^2 - y^2$$
$$c^2 = a^2 + b^2 - 2ay$$

Step 4
I wrote an equivalent equation that only used the variable y and simplified.

$$\cos C = \frac{y}{b}, \text{ so}$$
$$b \cos C = y$$

Step 5
I determined an equivalent expression for y.

$$c^2 = a^2 + b^2 - 2ay$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Step 6
I substituted the expression $b \cos C$ for y in my equation.

? How can you improve Heather's explanations in her proof of the cosine law?

A. Work with a partner to explain why she drew height AD in step 1.

B. In step 2, Heather created two different expressions that involved h^2. Explain how she did this.

C. Explain why she was able to set the expressions for h^2 equal in step 3.

D. In step 4, Heather eliminated the variable x. Explain how and why.

E. Explain how she determined an equivalent expression for y in step 5.

F. Explain why the final equation in step 6 is the most useful form of the cosine law.

Reflecting

G. Les wrote a similar proof, but he substituted $a - x$ for y instead of $a - y$ for x in the equation in step 3. How would his result differ from Heather's?

H. François started his proof by drawing a height from B to AC. How would this affect his final result?

I. **i)** Explain why you can use the cosine law to determine the unknown side q in $\triangle QRS$ on the previous page.

ii) Explain why you can use the cosine law to determine the unknown $\angle F$ in $\triangle DEF$ on the previous page.
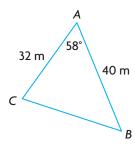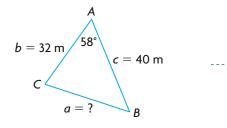
APPLY the Math

EXAMPLE 1 | Using reasoning to determine the length of a side

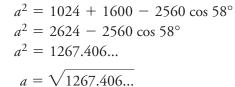Determine the length of *CB* to the nearest metre.

Justin's Solution

$a^2 = b^2 + c^2 - 2bc \cos A$
$a^2 = 32^2 + 40^2 - 2(32)(40) \cos 58°$

I labelled the sides with letters.
I couldn't use the sine law, because I didn't know a side length and the measure of its opposite angle.

I knew the lengths of two sides (*b* and *c*) and the measure of the contained angle between these sides ($\angle A$). I had to determine side *a*, which is opposite $\angle A$. I chose the form of the cosine law that includes these four values. Then I substituted the values I knew into the cosine law.

$a^2 = 1024 + 1600 - 2560 \cos 58°$
$a^2 = 2624 - 2560 \cos 58°$
$a^2 = 1267.406...$

$a = \sqrt{1267.406...}$
$a = 35.600...$

CB is 36 m.

Your Turn

After determining the length of *CB* in $\triangle ABC$ above, Justin used the sine law to determine that the measure of $\angle B$ is 50°, then concluded that the measure of $\angle C$ must be 72°. Use the cosine law to verify his solution for $\triangle ABC$.
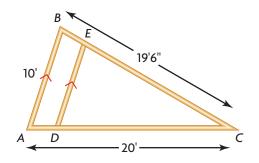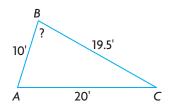
EXAMPLE **2**   Using reasoning to determine the measure of an angle

The diagram at the right shows the plan for a roof, with support beam *DE* parallel to *AB*. The local building code requires the angle formed at the peak of a roof to fall within a range of 70° to 80° so that snow and ice will not build up. Will this plan pass the local building code?

Luanne's Solution: Substituting into the cosine law, then rearranging

I drew a sketch, removing the support beam since it isn't needed to solve this problem.

The peak of the roof is represented by $\angle B$.

I labelled the sides I knew in the triangle.

I wrote all the lengths using the same unit, feet.

$a = 19.5$, $b = 20$, and $c = 10$

Since I only knew the lengths of the sides in the triangle, I couldn't use the sine law.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

I had to determine $\angle B$, so I decided to use the form of the cosine law that contained $\angle B$.

$$20^2 = 19.5^2 + 10^2 - 2(19.5)(10) \cos B$$
$$400 - 380.25 - 100 = -390 \, (\cos B)$$
$$-80.25 = -390 \, (\cos B)$$
$$\frac{-80.25}{-390} = \cos B$$
$$\cos^{-1}\left(\frac{80.25}{390}\right) = \angle B$$
$$78.125...° = \angle B$$

I substituted the side lengths into the formula and simplified.

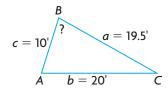I had to isolate cos B before I could determine $\angle B$.

The angle formed at the peak of the roof is 78°. This plan will pass the local building code.

My answer is reasonable because $\angle B$ should be the angle with the largest measure in the triangle.

78° lies within the acceptable range of 70° to 80°.

Emilie's Solution: Rearranging the cosine law before substituting

B
?   a = 19.5'
c = 10'
A   b = 20'   C

I drew a diagram, labelling the sides and angles. I wrote all the side lengths in terms of feet.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Since I wanted to determine $\angle B$ and I knew the length of all three sides, I wrote the form of the cosine law that contains $\angle B$.

$$b^2 + 2ac \cos B = a^2 + c^2 - 2ac \cos B + 2ac \cos B$$
$$b^2 + 2ac \cos B = a^2 + c^2$$
$$b^2 + 2ac \cos B - b^2 = a^2 + c^2 - b^2$$
$$2ac \cos B = a^2 + c^2 - b^2$$
$$\frac{2ac \cos B}{2ac} = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

I decided to rearrange the formula to solve for cos B by adding $2ac \cos B$ to both sides of the equation. Then I subtracted b^2 from both sides. Finally I divided both sides by $2ac$.

$$\cos B = \frac{19.5^2 + 10^2 - 20^2}{2(19.5)(10)}$$

I substituted the information that I knew into the rearranged formula and evaluated the right side.

$$\cos B = \frac{80.25}{390}$$
$$\cos B = 0.2057...$$

$$\angle B = \cos^{-1}(0.2057...)$$
$$\angle B = 78.125...°$$

The angle formed at the peak of the roof is 78°.
This plan passes the local building code.

I rounded to the nearest degree. The value of this angle is within the acceptable range.

Your Turn

a) Compare Luanne's Solution and Emilie's Solution. What are the advantages of each strategy?

b) Which strategy do you prefer for this problem? Explain.

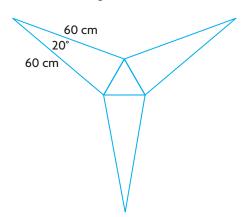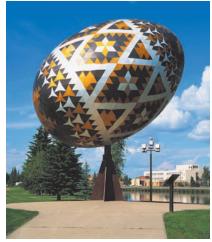**c)** Use your strategy and the cosine law to determine $\angle A$ in $\triangle ABC$ above.

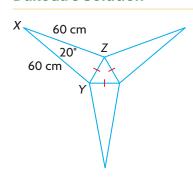EXAMPLE **3** | Solving a problem using the cosine law

A three-pointed star is made up of an equilateral triangle and three congruent isosceles triangles. Determine the length of each side of the equilateral triangle in this three-pointed star. Round the length to the nearest centimetre.

60 cm
20°
60 cm

The world's largest Ukrainian Easter egg (called a pysanka) is located in Vegreville, Alberta. It is decorated with 2208 equilateral triangles and 524 three-pointed stars.

Dakoda's Solution

X
60 cm
20° Z
60 cm
Y

I named the vertices of one of the isosceles triangles.

$(YZ)^2 = (XY)^2 + (XZ)^2 - 2(XY)(XZ) \cos (\angle YXZ)$
$(YZ)^2 = 60^2 + 60^2 - 2(60)(60) \cos 20°$
$(YZ)^2 = 3600 + 3600 - 6765.786...$

$(YZ)^2 = 434.213...$
$\quad YZ = \sqrt{434.213...}$
$\quad YZ = 20.837...$

Each side of the equilateral triangle has a length of 21 cm.

I knew two sides and the contained angle in each isosceles triangle, so I used the cosine law to write an equation that involved YZ. Then I substituted the information that I knew.
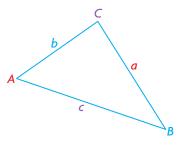
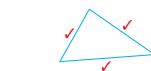3.3 Proving and Applying the Cosine Law **149**

In Summary

Key Idea

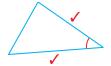- The cosine law can be used to determine an unknown side length or angle measure in an acute triangle.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Need to Know

- You can use the cosine law to solve a problem that can be modelled by an acute triangle when you know:
  - two sides and the contained angle.
  - all three sides.
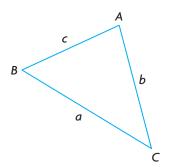
- The contained angle is the angle between two known sides.
- When using the cosine law to determine an angle, you can:
  - substitute the known values first, then solve for the unknown angle.
  - rearrange the formula to solve for the cosine of the unknown angle, then substitute and evaluate.

CHECK Your Understanding

1. Suppose that you are given each set of data for $\triangle ABC$. Can you use the cosine law to determine c? Explain.

   a) $a = 5$ cm, $\angle A = 52°$, $\angle C = 43°$

   b) $a = 7$ cm, $b = 5$ cm, $\angle C = 43°$
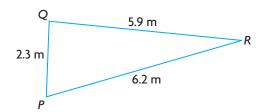
2. Determine the length of side x to the nearest centimetre.
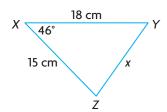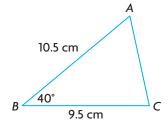

3. Determine the measure of $\angle P$ to the nearest degree.
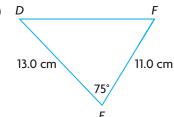
PRACTISING

4. Determine each unknown side length to the nearest tenth of a centimetre.

a)
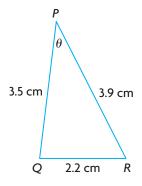
b)
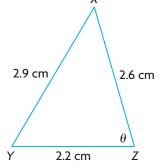
5. Determine the measure of each indicated angle to the nearest degree.

a)

b)

6. Sketch each triangle, based on the given equation. Then solve for the unknown side length or angle measure. Round all answers to the nearest tenth of a unit.

a) $w^2 = 15^2 + 16^2 - 2(15)(16) \cos 75°$

b) $k^2 = 32^2 + 35^2 - 2(32)(35) \cos 50°$

c) $48^2 = 46^2 + 45^2 - 2(46)(45) \cos Y$

d) $13^2 = 17^2 + 15^2 - 2(17)(15) \cos G$

3.3 Proving and Applying the Cosine Law

7. Solve each triangle. Round all answers to the nearest tenth of a unit.

  a) In $\triangle DEF$, $d = 5.0$ cm, $e = 6.5$ cm, and $\angle F = 65°$.

  b) In $\triangle PQR$, $p = 6.4$ m, $q = 9.0$ m, and $\angle R = 80°$.

  c) In $\triangle LMN$, $l = 5.5$ cm, $m = 4.6$ cm, and $n = 3.3$ cm.

  d) In $\triangle XYZ$, $x = 5.2$ mm, $y = 4.0$ mm, and $z = 4.5$ mm.

8. The pendulum of a grandfather clock is 100.0 cm long. When the pendulum swings from one side to the other side, the horizontal distance it travels is 9.6 cm.

  a) Draw a diagram of the situation.

  b) Determine the angle through which the pendulum swings. Round your answer to the nearest tenth of a degree.

9. Determine the perimeter of $\triangle SRT$, if $\angle S = 60°$, $r = 15$ cm, and $t = 20$ cm. Round your answer to the nearest tenth of a centimetre.

10. A parallelogram has sides that are 8 cm and 15 cm long. One of the angles in the parallelogram measures 70°. Explain how you could determine the length of the shorter diagonal.

11. a) A clock has a minute hand that is 20 cm long and an hour hand that is 12 cm long. Determine the distance between the tips of the hands at

  i) 2:00.   **ii)** 10:00.

  b) Discuss your results for part a).

12. Emilie makes stained glass windows to sell at the Festival du Bois in Maillardville, British Columbia. Each piece of glass is surrounded by lead edging. Emilie claims that she can create an acute triangle in part of a window using pieces of lead that are 15 cm, 36 cm, and 60 cm. Is she correct? Justify your decision.

The Festival du Bois, one of British Columbia's greatest celebrations of French-Canadian culture, is held in March.
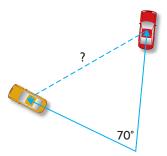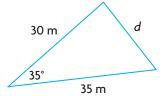
13. Two drivers leave their school at the same time and travel on straight roads that diverge by 70°. One driver travels at an average speed of 33.0 km/h. The other driver travels at an average speed of 45.0 km/h. How far apart will the two drivers be after 45 min?
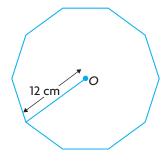
Closing

14. Use the triangle at the right to create a problem that involves side lengths and interior angles. Then describe how to determine the length of side *d*. Exchange your problem with a classmate.

Extending

15. The distance from the centre, *O*, of a regular decagon to each vertex is 12 cm. Determine the area of the decagon. Round your answer to the nearest square centimetre.

16. The centre, *O*, of a regular pentagon is a perpendicular distance of 1.5 cm from each side. Determine the perimeter and area of the pentagon.

17. An ulu is an Inuit all-purpose knife, traditionally used by women. The metal blade of one type of ulu is roughly triangular in shape, with the cutting edge opposite the vertex where the handle is attached. The other sides of the ulu are roughly equal. Describe a functional ulu that has a 14 cm blade, measured point to point. Include the vertex angle at the handle and the side lengths in your description.

Solving Problems Using Acute Triangles

GOAL

Solve problems using the primary trigonometric ratios and the sine and cosine laws.

EXPLORE...

- Two planes leave an airport on different runways at the same time. One heads S40°W and the other heads S60°E. Create a problem about the planes that can be solved only by using the cosine law. Solve the problem.
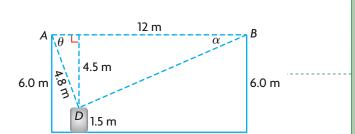
LEARN ABOUT the Math

Two security cameras in a museum must be adjusted to monitor a new display of fossils. The cameras are mounted 6 m above the floor, directly across from each other on opposite walls. The walls are 12 m apart. The fossils are displayed in cases made of wood and glass. The top of the display is 1.5 m above the floor. The distance from the camera on the left to the centre of the top of the display is 4.8 m. Both cameras must aim at the centre of the top of the display.

? What is the angle of depression for each camera?

EXAMPLE 1 Connecting an acute triangle model to a situation

Determine the angles of depression, to the nearest degree, for each camera.

Vlad's Solution: Using primary trigonometric ratios and the cosine law

I drew a diagram. I placed the cameras 6 m from the floor, 12 m away from each other on opposite walls at points A and B. I wasn't sure where to place the display or its centre, D. The display had to be closer to camera A since the distance from camera A to the display was only 4.8 m. Subtracting the display height from 6 m gave me the distance from the display to the horizontal between the cameras. I labelled the angles of depression using θ and α.
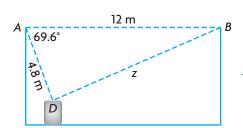
$$\sin \theta = \frac{4.5}{4.8}$$

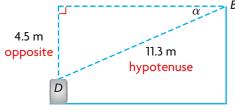$$\theta = \sin^{-1}\left(\frac{4.5}{4.8}\right)$$

$$\theta = 69.635...°$$

In the right triangle containing angle θ, the side of length 4.5 m is opposite angle θ, and the side of length 4.8 m is the hypotenuse. I could use the sine ratio to determine the measure of θ.
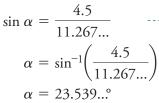
I knew the lengths of two sides in $\triangle ABD$ and the angle between them. So, I was able to use the cosine law to determine the length of the side opposite camera A, which I labelled z.

$$z^2 = 12^2 + 4.8^2 - 2(12)(4.8)\cos 69.635...°$$
$$z^2 = 126.952...$$
$$z = \sqrt{126.952...}$$
$$z = 11.267...$$

$$\sin \alpha = \frac{4.5}{11.267...}$$
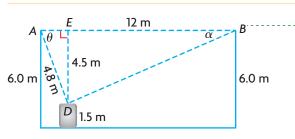
Based on the sides I knew in the right triangle containing angle α, I wrote an equation using the sine ratio.

$$\alpha = \sin^{-1}\left(\frac{4.5}{11.267...}\right)$$

$$\alpha = 23.539...°$$

To monitor the display effectively, camera A must be adjusted to an angle of depression of 70° and camera B must be adjusted to an angle of depression of 24°.

Michel's Solution: Using only primary trigonometric ratios

I drew a diagram by placing the cameras at points A and B and the centre of the display at point D. I knew point D had to be closer to camera A because the distance between it and camera B had to be greater than 4.8 m. Subtracting the display height from the camera height gave me the length of DE, the height of $\triangle ABD$. I labelled the angles of depression θ and α.
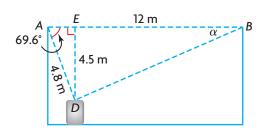
$$\sin \theta = \frac{4.5}{4.8}$$

$$\theta = \sin^{-1}\left(\frac{4.5}{4.8}\right)$$

$$\theta = 69.635...°$$

In right triangle *ADE*, *DE* is opposite angle θ and *AD* is the hypotenuse. Since I knew the lengths of both sides, I used the sine ratio to determine angle θ.
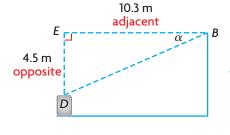
$$\cos 69.635...° = \frac{AE}{4.8}$$

$$4.8 \left(\cos 69.635...°\right) = AE$$

$$1.670... = AE$$

In right triangle *ADE*, *AE* is adjacent to angle θ and *AD* is the hypotenuse. Since I knew the length of *AD* and the measure of angle θ, I used the cosine ratio to determine the length of *AE*.

$$EB = 12 - 1.670...$$

$$EB = 10.329...$$

To determine the length of *EB*, I subtracted the length of *AE* from 12.

Based on the sides I knew in right triangle *DEB*, I wrote an equation using the tangent ratio to determine angle α.

$$\tan \alpha = \frac{4.5}{10.329...}$$

$$\alpha = \tan^{-1}\left(\frac{4.5}{10.329...}\right)$$

$$\alpha = 23.539...°$$

Camera A must be adjusted to an angle of depression of 70° and camera B must be adjusted to an angle of depression of 24° to ensure that they both point to the centre of the display.

Reflecting

A. Why do you think Vlad started his solution by using the right triangle that contained angle θ instead of the right triangle that contained angle α?

B. Could Vlad have determined the value of α using the sine law? Explain.

C. Which solution do you prefer? Justify your choice.
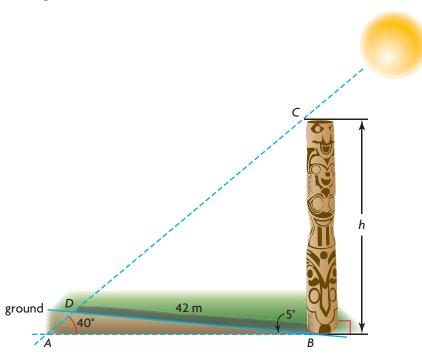
APPLY the Math

EXAMPLE 2 | **Connecting acute triangle models to indirect measurement**

The world's tallest free-standing totem pole is located in Beacon Hill Park in Victoria, British Columbia. It was carved from a single cedar log by Chief Mungo Martin, noted Kwakiutl (Kwakwaka'wakw) carver, with a team that included his son David and Henry Hunt. It was erected in 1956.

While visiting the park, Manuel wanted to determine the height of the totem pole, so he drew a sketch and made some measurements:

- I walked along the shadow of the totem pole and counted 42 paces, estimating each pace was about 1 m.
- I estimated that the **angle of elevation** of the Sun was about 40°.
- I observed that the shadow ran uphill, and I estimated that the angle the hill made with the horizontal was about 5°.

How can Manuel determine the height of the totem pole to the nearest metre?

7152_FM11SB_C3_L3_ALB_pp154-165.indd   157   9/23/10   4:32:32 PM

Manuel's Solution

h is the height of $\triangle ABC$, but it is also a side in acute triangle DCB.

> I needed to determine the angles in this triangle to be able to determine h using the sine law.

$\angle ADB = 180° - 40° - 5°$
$\angle ADB = 135°$

> I used the two angles I knew in $\triangle ADB$ to determine the third angle in this triangle, since the angles in a triangle add to 180°.
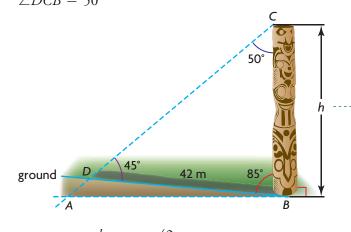
$\angle CDB = 180° - 135°$
$\angle CDB = 45°$

> I subtracted $\angle ADB$ from 180° to determine $\angle CDB$, since these are supplementary angles.

$\angle CBD = 90° - 5°$
$\angle CBD = 85°$

> I subtracted $\angle DBA$ from 90° to determine $\angle CBD$, since these are complementary angles.

$\angle DCB = 180° - 45° - 85°$
$\angle DCB = 50°$

> I used the two angles I determined in $\triangle DCB$ to determine the third angle in this triangle.

> I added all the information I determined about $\triangle DCB$ to my sketch.

$$\frac{h}{\sin 45°} = \frac{42}{\sin 50°}$$

> I used the sine law to write an equation that contained h. Then I solved for h.

$$\sin 45°\left(\frac{h}{\sin 45°}\right) = \sin 45°\left(\frac{42}{\sin 50°}\right)$$

$$h = \sin 45°\left(\frac{42}{\sin 50°}\right)$$

$$h = 38.768...$$

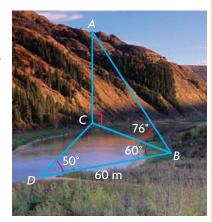The totem pole is 39 m tall.

Your Turn

List some sources of error that may have occurred in Manuel's strategy that would affect the accuracy of his determination.

EXAMPLE **3**

Solving a three-dimensional problem

Brendan and Diana plan to climb the cliff at Dry Island Buffalo Jump, Alberta. They need to know the height of the climb before they start. Brendan stands at point *B*, as shown in the diagram. He uses a clinometer to determine $\angle ABC$, the angle of elevation to the top of the cliff. Then he estimates $\angle CBD$, the angle between the base of the cliff, himself, and Diana, who is standing at point *D*. Diana estimates $\angle CDB$, the angle between the base of the cliff, herself, and Brendan.

Determine the height of the cliff to the nearest tenth of a metre.

Diana's Solution

In $\triangle DBC$,
$\angle BCD = 180° - 60° - 50°$
$\angle BCD = 70°$

> I didn't have enough information about $\triangle ABC$ to determine the height, *AC*. I needed the length of *BC*. *BC* is in $\triangle ABC$, but it is also in $\triangle DBC$.
>
> I knew two angles and a side length in $\triangle DBC$. Before I could determine *BC*, I had to determine $\angle BCD$. I used the fact that the sum of all three interior angles is 180°.

$$\frac{BC}{\sin D} = \frac{BD}{\sin C}$$

$$\frac{BC}{\sin 50°} = \frac{60}{\sin 70°}$$

> I used the sine law to write an equation that involved *BC* in $\triangle DBC$.

$$\sin 50°\left(\frac{BC}{\sin 50°}\right) = \sin 50°\left(\frac{60}{\sin 70°}\right)$$

> To solve for *BC*, I multiplied both sides of the equation by sin 50°.

$$BC = \sin 50°\left(\frac{60}{\sin 70°}\right)$$

$$BC = 48.912...$$

$$\tan 76° = \frac{AC}{BC}$$

$$\tan 76° = \frac{AC}{48.912...}$$

> I knew that $\triangle ABC$ is a right triangle. I also knew that *AC* is opposite the 76° angle and *BC* is adjacent to it. So, I used the tangent ratio to write an equation that involved *AC*.

$$48.912...\,(\tan 76°) = 48.912...\left(\frac{AC}{48.912...}\right)$$

$$196.177... = AC$$

The height of the cliff is 196.2 m.

Your Turn

Create a three-dimensional problem that can be solved using Diana's strategy. What features of your problem make it necessary to use two triangles to solve the problem?

In Summary

Key Idea

- The sine law, the cosine law, the primary trigonometric ratios, and the sum of angles in a triangle may all be useful when solving problems that can be modelled using acute triangles.

Need to Know

- To decide whether you need to use the sine law or the cosine law, consider the information given about the triangle and the measurement to be determined.

Information Given	Measurement to be Determined	Use
two sides and the angle opposite one of the sides	angle	sine law
two angles and a side	side	sine law
two sides and the contained angle	side	cosine law
three sides	angle	cosine law

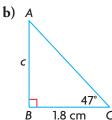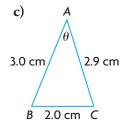- Drawing a clearly labelled diagram makes it easier to select a strategy for solving a problem.

CHECK *Your Understanding*

1. Explain how you would determine the indicated angle measure or side length in each triangle.

   a)

   b) A

   c)
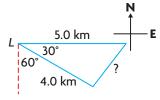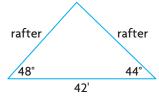
2. a) Use the strategies you described to determine the measurements indicated in question 1. Round your answers to the nearest tenth of a unit.

   b) Compare your answers for questions 1 and 2a) with a classmate's answers. Which strategy seems to be most efficient for each?

PRACTISING
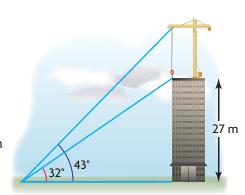
3. A kayak leaves Rankin Inlet, Nunavut, and heads due east for 5.0 km, as shown in the diagram. At the same time, a second kayak travels in a direction S60°E from the inlet for 4.0 km. How far apart, to the nearest tenth of a kilometre, are the kayaks?

   a) Describe how you can solve the problem.

   b) Determine the distance between the kayaks.

4. How long, to the nearest inch, is each rafter in the roof shown?
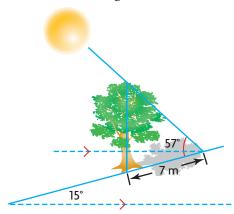
5. A crane stands on top of a building, as shown.

   a) How far is the point on the ground from the base of the building, to the nearest tenth of a metre?

   b) How tall is the crane?

6. A tree is growing on a hillside, as shown. The hillside is inclined at an angle of 15° to the horizontal. The tree casts a shadow uphill. How tall is the tree, to the nearest metre?
  a) Describe how you can solve the problem.
  b) Determine the height of the tree.

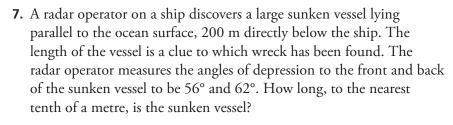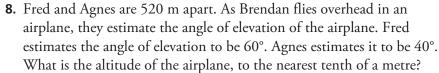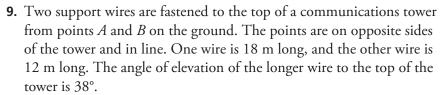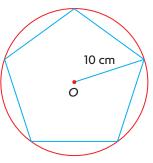**7.** A radar operator on a ship discovers a large sunken vessel lying parallel to the ocean surface, 200 m directly below the ship. The length of the vessel is a clue to which wreck has been found. The radar operator measures the angles of depression to the front and back of the sunken vessel to be 56° and 62°. How long, to the nearest tenth of a metre, is the sunken vessel?

8. Fred and Agnes are 520 m apart. As Brendan flies overhead in an airplane, they estimate the angle of elevation of the airplane. Fred estimates the angle of elevation to be 60°. Agnes estimates it to be 40°. What is the altitude of the airplane, to the nearest tenth of a metre?

9. Two support wires are fastened to the top of a communications tower from points *A* and *B* on the ground. The points are on opposite sides of the tower and in line. One wire is 18 m long, and the other wire is 12 m long. The angle of elevation of the longer wire to the top of the tower is 38°.
  a) How tall is the tower, to the nearest tenth of a metre?
  b) How far apart are points *A* and *B*, to the nearest tenth of a metre?

10. A regular pentagon is inscribed in a circle with centre *O*, as shown in the diagram.
  a) Work with a partner to develop a strategy to determine the perimeter of the pentagon.
  b) Carry out your strategy to determine the perimeter to the nearest tenth of a centimetre.

Ships are sometimes deliberately sunk (scuttled) to form breakwaters and artificial reefs.

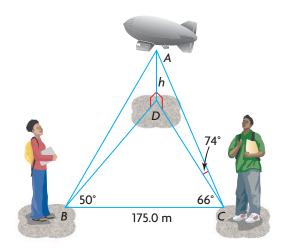**11.** Ryan is in a police helicopter, 400 m directly above the Sea to Sky highway near Whistler, British Columbia. When he looks north, the angle of depression to a car accident is 65°. When he looks south, the angle of depression to the approaching ambulance is 30°.

    a) How far away is the ambulance from the scene of the accident, to the nearest tenth of a metre?

    b) The ambulance is travelling at 80 km/h. How long will it take the ambulance to reach the scene of the accident?

12. The radar screen in the air-traffic control tower at the Edmonton International Airport shows that two airplanes are at the same altitude. According to the range finder, one airplane is 100 km away, in the direction N60°E. The other airplane is 160 km away, in the direction S50°E.

    a) How far apart are the airplanes, to the nearest tenth of a kilometre?

    b) If the airplanes are approaching the airport at the same speed, which airplane will arrive first?

13. In a parallelogram, two adjacent sides measure 10 cm and 12 cm. The shorter diagonal is 15 cm. Determine, to the nearest degree, the measures of all four angles in the parallelogram.

14. Two students decided to determine the altitude, h, of a promotional blimp flying over McMahon Stadium in Calgary. The students' measurements are shown in the diagram. Determine h to the nearest tenth of a metre. Explain each of your steps.
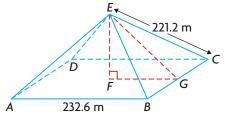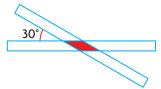
Closing

15. Create a real-life problem that can be modelled by an acute triangle. Exchange the problem you created with a classmate. Sketch the situation in your classmate's problem, and explain what must be done to solve it.

Extending

16. The Great Pyramid at Giza in Egypt has a square base with sides that are 232.6 m long. The distance from the top of the pyramid to each corner of the base was originally 221.2 m.

  a) Determine the apex angle of a face of the pyramid (for example, $\angle AEB$) to the nearest degree.

  b) Determine the angle that each face makes with the base (for example, $\angle EGF$) to the nearest degree.

17. Cut out two paper strips, each 5 cm wide. Lay them across each other as shown at the right. Determine the area of the overlapping region. Round your answer to the nearest tenth of a square centimetre.

History | Connection

Fire Towers and Lookouts

For about 100 years, observers have been watching for forest fires. Perched in lookouts on high ground or on tall towers (there are three fire towers in Alberta that are 120 m high), the observers watch for smoke in the surrounding forest. When the observers see signs of a fire, they report the sighting to the Fire Centre.

The first observers may have had as little as a map, binoculars, and a horse to ride to the nearest station to make a report. Today, observers have instruments called alidades and report using radios. Alidades consist of a local map, fixed in place, with the tower or lookout at the centre of the map. Compass directions are marked in degrees around the edge of the map. A range finder rotates on an arc above the map. The observer rotates the range finder to get the smoke in both sights, notes the direction, and reports it to the Fire Centre. A computer at the Fire Centre can use the locations of the fire towers and directions from two or more observers to fix the location of the fire. The latitude and longitude are then entered into a helicopter's flight GPS system. The helicopter's crew flies to the fire, records a more accurate location, and reports on the fire.

A. How could a Fire Centre use trigonometry to determine the location of a fire?

B. How could an observer use trigonometry to estimate the size of a fire?
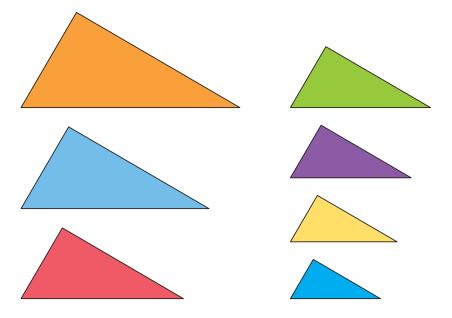
Applying Problem-Solving Strategies

Analyzing a Trigonometry Puzzle

Puzzles do not always have precise solutions. They cannot always be solved purely by deduction, although logic helps.

The Puzzle

A. Below are seven similar right triangles. Trace the triangles, and cut them out.

B. Use all seven triangles to form a single square, with no overlapping.

C. If the hypotenuse of the greatest triangle is 10 units long, what is the area of the square?

The Strategy

D. Describe the strategy you used to form the square.

E. Describe the strategy you used to determine the area of the square.

1. Determine the indicated side length or angle measure in each triangle. Round answers to the nearest tenth of a unit.

    a)

    A
    82° 4.1 cm
    C
    θ 6.0 cm
    B
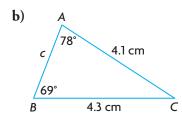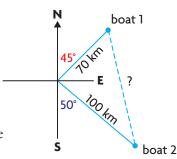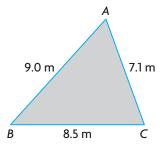
    b)

    A
    78° 4.1 cm
    c
    69°
    B 4.3 cm C

2. In $\triangle PQR$, $\angle P = 80°$, $\angle Q = 48°$, and $r = 20$ cm. Solve $\triangle PQR$. Round answers to the nearest tenth of a unit.

3. The radar screen of a Coast Guard rescue ship shows that two boats are in the area, as shown in the diagram. How far apart are the two boats, to the nearest tenth of a kilometre?

    N
    boat 1
    45° 70 km
    W —————— E ?
    50° 100 km
    S boat 2

4. A parallelogram has adjacent sides that are 11.0 cm and 15.0 cm long. The angle between these sides is 50°. Determine the length of the shorter diagonal to the nearest tenth of a centimetre.

5. Points P and Q lie 240 m apart in line with and on opposite sides of a communications tower. The angles of elevation to the top of the tower from P and Q are 50° and 45°, respectively. Determine the height of the tower to the nearest tenth of a metre.

6. Terry is designing a triangular patio, as shown. Determine the area of the patio to the nearest tenth of a square metre.

    A
    9.0 m 7.1 m
    B 8.5 m C

7. In an acute triangle, two sides are 2.4 cm and 3.6 cm. One of the angles is 37°. How can you determine the third side in the triangle? Explain.

8. Why do you need both the sine law and the cosine law to determine side lengths in an acute triangle?

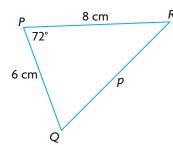*WHAT DO You Think Now?* Revisit **What Do You Think?** on page 129. How have your answers and explanations changed?

FREQUENTLY ASKED *Questions*

Q: **To use the cosine law, what do you need to know about a triangle?**

A: You need to know two sides and the contained angle, or three sides in the triangle.
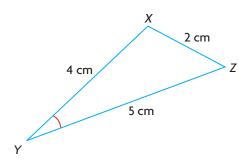
For example, you can use the cosine law to determine the length of p.

You know:
- the lengths of two sides.
- the measure of the contained angle.

You can use the cosine law to determine the length of the side opposite the contained angle, p.

$p^2 = 8^2 + 6^2 - 2(8)(6) \cos 72°$

Solving for p will determine the length.

You can also use the cosine law to determine the measure of $\angle Y$.

You know:
- the lengths of all three sides.

You can use the cosine law to determine the measure of $\angle Y$.

$2^2 = 4^2 + 5^2 - 2(4)(5) \cos Y$

$$\frac{2^2 - 4^2 - 5^2}{-2(4)(5)} = \cos Y$$

Solving for Y will determine the measure of $\angle Y$.

Q: **When solving a problem that can be modelled by an acute triangle, how do you decide whether to use the primary trigonometric ratios, the sine law, or the cosine law?**

A: Draw a clearly labelled diagram of the situation to record what you know.
- You may be able to use a primary trigonometric ratio if the diagram involves a right triangle.
- Use the sine law if you know two sides and one opposite angle, or two angles and one opposite side.
- Use the cosine law if you know all three sides, or two sides and the angle between them.

You may need to use more than one strategy to solve some problems.

PRACTISING

Lesson 3.1

1. Jane claims that she can draw an acute triangle using the following information: $a = 6$ cm, $b = 8$ cm, $c = 10$ cm, $\angle A = 30°$, and $\angle B = 60°$. Is she correct? Explain.

2. Which of the following are not correct for acute triangle DEF?

   a) $\dfrac{d}{\sin D} = \dfrac{f}{\sin F}$ c) $f \sin E = e \sin F$
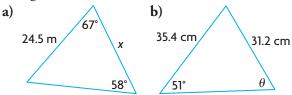
   b) $\dfrac{\sin E}{e} = \dfrac{\sin D}{d}$ d) $\dfrac{d}{\sin D} = \dfrac{\sin F}{f}$

Lesson 3.2
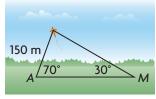
3. Determine the indicated side or angle in each triangle to the nearest tenth of a unit.

   a)

   67°
   24.5 m x
   58°

   b)

   35.4 cm 31.2 cm
   51° θ

4. Solve $\triangle ABC$, if $\angle A = 75°$, $\angle B = 50°$, and the side between these angles is 8.0 cm. Round answers to the nearest tenth of a unit.

5. Allison is flying a kite. She has released the entire 150 m ball of kite string. She notices that the string forms a 70° angle with the ground. Marc is on the other side of the kite and sees the kite at an angle of elevation of 30°. How far is Marc from Allison, to the nearest tenth of a metre?
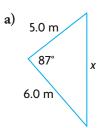
   150 m
   70° 30°
   A M

Lesson 3.3

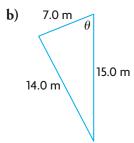6. Which of these is not a form of the cosine law for $\triangle ABC$? Explain.

   a) $a^2 = b^2 + c^2 - 2bc \cos B$
   b) $c^2 = a^2 + b^2 - 2ab \cos C$
   c) $b^2 = a^2 + c^2 - 2ac \cos B$

7. Determine the indicated side or angle. Round answers to the nearest tenth of a unit.

   a)

   5.0 m
   87° x
   6.0 m

   b)

   7.0 m
   θ
   14.0 m 15.0 m

8. Solve $\triangle ABC$, if $\angle A = 58°$, $b = 10.0$ cm, and $c = 14.0$ cm. Round answers to the nearest tenth of a unit.

9. Two airplanes leave the Hay River airport in the Northwest Territories at the same time. One airplane travels at 355 km/h. The other airplane travels at 450 km/h. About 2 h later, they are 800 km apart. Determine the angle between their paths, to the nearest degree.

Lesson 3.4

10. From a window in an apartment building, the angle of elevation to the top of a flagpole across the street is 9°. The angle of depression is 22° to the base of the flagpole. How tall is the flagpole, to the nearest tenth of a metre?

    9°
    22°
    8 m

11. A bush pilot delivers supplies to a remote camp by flying 255 km in the direction N52°E. While at the camp, the pilot receives a radio message to pick up a passenger at a village. The village is 85 km S21°E from the camp. What is the total distance, to the nearest kilometre, that the pilot will have flown by the time he returns to his starting point?

12. A canoeist starts from a dock and paddles 2.8 km N34°E. Then she paddles 5.2 km N65°W. What distance, and in which direction, should a second canoeist paddle to reach the same location directly, starting from the same dock? Round all answers to the nearest tenth of a unit.

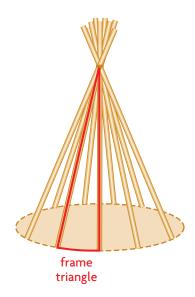## Acute Triangles in First Nations and Métis Cultures

Inside a Dakota tipi

Blackfoot tipis

Structures made from acute triangles are used in most cultures. The First Nations and Métis peoples who lived on the Prairies used acute-triangle structures to cook, dry meat, and transport goods and people. Acute triangles are also the support for tipis. Tipis suit a life based on the migration of the buffalo, because they are easily assembled and taken down. Tipis are still used for shelter on camping trips and for ceremonial purposes.

The frame of a tipi consists of wooden poles supported by an inner tripod. Additional poles are laid in and tied with rope. When the frame is complete, the covering is drawn over the frame using two additional poles. The interior of the tipi creates a roughly conical shape. The number of poles used is indicative of geographic area and people, ranging from 13 to 21 poles.

? **What are the measures of all the sides and angles in each triangle in the frame of a tipi?**

A. Design a tipi. Choose the number of poles, the interior height, and the diameter of the base.

B. What regular polygon forms the base of your tipi? What type of triangle is formed by the ground and supporting poles? Explain how you know.

C. Determine the side lengths of the frame triangles.

D. Determine the interior angles of the frame triangles.

E. Explain why your design is functional.

frame triangle

Task | *Checklist*

✔ Did you draw labelled diagrams for the problem?

✔ Did you show your work?

✔ Did you provide appropriate reasoning?

✔ Did you explain your thinking clearly?

Project Connection

Creating Your Research Question or Statement

A well-written research question or statement clarifies exactly what your project is designed to do. It should have the following characteristics:

- The research *topic* is easily identifiable.
- The *purpose* of the research is clear.
- The question/statement is *focused*. The people who are listening to or reading the question/statement will know what you are going to be researching.

A good question requires thought and planning. Below are three examples of initial questions or statements and how they were improved.

Unacceptable Question/Statement	Why?	Acceptable Question/Statement
Is mathematics used in computer technology?	too general	What role has mathematics played in the development of computer animation?
Water is a shared resource.	too general	Homes, farms, ranches, and businesses east of the Rockies all use runoff water. When there is a shortage, that water must be shared.
Do driver's education programs help teenagers parallel park?	too specific, unless you are going to generate your own data	Do driver's education programs reduce the incidence of parking accidents?

Evaluating Your Research Question or Statement

You can use the following checklist to determine if your research question/statement is effective.

1. Does the question/statement clearly identify the main objective of the research? After you read the question/statement to a few classmates, can they tell you what you will be researching?
2. Are you confident that the question/statement will lead you to sufficient data to reach a conclusion?
3. Is the question/statement interesting? Does it make you want to learn more?
4. Is the topic you chose purely factual, or are you likely to encounter an issue, with different points of view?

Sarah chose the changes in population of the Western provinces and the territories over the last century as her topic. Below, she describes how she determined that her research question for this topic is effective.

Sarah's Question

My question is, "Which Western province or territory grew the fastest over the last century and why?" I will use 1900 to 2000 as the time period.

I evaluated my question using the research question checklist, and I feel that it is a good one. Here is why:

1. My question tells what I plan to do: I read my question to three friends, and they all described what I had in mind.

2. I am confident that there is a lot of data available on populations, and that there is a lot of historical information available on why the populations changed.

3. I'm really interested in history, but I don't know enough about how the West and North grew and why. I'll find out lots of new things. Whatever I find out should also interest some of my classmates, as it's about where we live.

4. I expect that I will find several different points of view on why populations grew, and I hope that I will be able to conclude which factors were most important.

Chinese immigrants and workers arrive at William Head Quarantine Station, British Columbia, in 1917. What factors affected where they chose to settle?

This photograph shows a potash mine in Saskatchewan. How do resources affect population growth?

Your Turn

A. Write a research question for your topic.

B. Use the checklist to evaluate your question. Adjust your question as needed.

C. Make an appointment to discuss the pros and cons of your research question with your teacher. Be prepared to discuss your plan for collecting the data you will need to come to a conclusion. Adjust your question as needed.